Cost Risk Analysis

CON 270
Intermediate Cost & Price Analysis
Evaluating the Pricing Position

As I understand it, in the early 90’s the automotive industry was tasked with developing lead-free paints. My 92’ car was apparently not one of the new painting process success stories given that the paint on my car began to delaminate in a number of areas. This led me to get an estimate for having my car repainted from the local auto paint shop. The initial estimate was about $1100 which came as something of a shock given all of the advertisements you see on TV for auto painting for $199 to $299. Since I was not familiar with auto painting I asked the painter what the process entailed. He outlined the hours required to: prep the car (taping, removal of emblems, etc.); strip the paint; apply the primer and the finishing coats; and replace emblems, pin-stripping, rubber strips, etc. Sensing my apprehension about the price, he suggested that he might be able to get the price down to $800 if he was able to: reuse the emblems and rubber strips; leave off the pin-stripping; cut some time in prep; and if some other steps went better than expected. I followed up with my mechanic, who was familiar with auto painting costs, and he stated that it would be reasonable to pay $1100-$1200 for this work.

How did having all of this information help? It provided me a better understanding of the requirements, some grounds for determining if the initial quote was realistic, and also some insight into the risk associated with the $800 and $1100 quotes. Two things became apparent from our discussion: the $199 to $299 advertisements could not possibly entail the same level of work as the $1100 effort; and there was a greater risk and a lower probability of success in performing the work for $800 than for the $1100.

Perhaps in this example you can see some of the concepts we have discussed in the course. There is market research, information other than certified cost and pricing data, interviewing experts, and from our statistics block, the idea that with every population (in this case, auto paint prices) there is a distribution with some degree of dispersion. I would like to suggest that these concepts could assist us in performing cost realism and price analysis.

Cost realism analysis is the process of independently reviewing and evaluating specific elements of each offeror’s proposed cost estimate to determine whether the estimated proposed cost elements are realistic for the work to be performed; reflect a clear understanding of the requirements; and are consistent with the unique methods of performance and materials described in the offeror’s technical proposal. Price analysis is the process of examining and evaluating a proposed price without evaluating its separate cost elements and proposed profit.
**Price Analysis Example:** Let’s say that we are using a time and materials contract and that through further discussions we determine the most optimistic estimate to paint the car is $700, the most probable is $1100, and the most pessimistic is $1500. Assuming a triangular distribution, we could conclude an estimate of $1100 would represent a 50% chance of over-running and a 50% chance of under-running. You can see that if we use the $800 estimate we approach a 100% chance of over-running while estimates closer to $1500 reduce the risk of over-run.

![Triangle Diagram]

**Cost Analysis Example #1:** We are purchasing the painting services on a time and materials contract for which the contractor has proposed a price of $935. We have asked some fact-finding questions and have gained the following insights into the cost elements associated with painting a car. We arrived at these assumptions through discussions with the contractor much like a government IPT or a government/contractor IPT might work.

**Labor:** Most likely it will take 22 man-hours to paint the car. The most pessimistic estimate is 32 hours and most optimistic is 18 hours. The contractor assumed it would require 22 hours.

**Fully Burdened Labor Rate:** The contractor has a billing rate of $32.50 an hour.

**Materials:** Depending on reuse of the existing car parts, the material costs could range anywhere from $150 to $350. Any cost within that range is just as likely to occur. The contractor believes that it would be reasonable to expect to spend $200 based on an assumption that most of the parts could be reused.

**Material Mark-Up Rate:** If the contractor can purchase the materials wholesale the rate is 10%. If instead the materials have to be purchased through a dealership then the rate is 20%. There is a 25% probability that the parts can be purchased wholesale. The contractor expects to purchase the parts wholesale.
CON 270, Intermediate Cost and Price Analysis

**Contractor's Estimate:**

<table>
<thead>
<tr>
<th>Labor Hours</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Billing Rate</td>
<td>$32.50</td>
</tr>
<tr>
<td>Subtotal Labor</td>
<td>$715</td>
</tr>
<tr>
<td>Material Cost</td>
<td>$200</td>
</tr>
</tbody>
</table>

As it is, this point estimate does not communicate the uncertainty inherent in the estimate nor does it provide the decision-maker with any measure of the risk associated with the $935 position. Now, what if we incorporate our assumptions:

**Labor Hours:**

18 22 32

**Materials:**

$150 $350

**Material Rate:**

PROBABILITY

75%

MH RATE

10% 20%

25%
Now if we were to randomly sample from each of these distributions 1000 times, each time generating an estimate at total price, we would generate a total price distribution something like this:
Where does the contractor’s position of $935 fall on this distribution?

As you can see from the distribution, the area to the left of $935 is about 13% and the area to the right of $935 is about 87%. In other words, there is a 87% probability that the price for the effort will actually be more than $935. In contrast, if we wanted a 50/50 probability of success, a reasonable price would be somewhere in the neighborhood of $1100. Since with this contracting arrangement, the government for all intents and purposes, will basically pay the actual cost, there is no risk to the contractor in under-bidding the effort.

Cost Analysis Example #2: In this case, rather than a time and materials contract, let’s assume that we have a firm fixed price contract for which the contractor has proposed a price of $1330. Again, we will review our assumptions.
Labor: Most likely it will take 22 man-hours to paint the car. The most pessimistic estimate is 32 hours and most optimistic is 18 hours. The contractor assumed it would require 28 hours.

Fully Burdened Labor Rate: The contractor has a billing rate of $32.50 an hour.

Materials: Depending on reuse of the existing car parts, the material costs could range anywhere from $150 to $350. Any cost within that range is just as likely to occur. The contractor believes that it would be reasonable to expect to spend $350 based on an assumption that none of the parts could be reused.

Material Mark-Up Rate: If the contractor can purchase the materials wholesale the rate is 10%. If instead the materials have to be purchased through a dealership then the rate is 20%. There is a 25% probability that the parts can be purchased wholesale. The contractor expects to purchase the parts through a dealership.

Contractor’s Estimate:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor Hours</td>
<td>28</td>
</tr>
<tr>
<td>Billing Rate</td>
<td>$32.50</td>
</tr>
<tr>
<td>Subtotal Labor</td>
<td>$910</td>
</tr>
<tr>
<td>Material Cost</td>
<td>$350</td>
</tr>
</tbody>
</table>
Where does the contractor’s position of $1330 fall on this distribution?

As you can see from the distribution, the area to the left of $1330 is about 95% and the area to the right of $1330 is about 5%. In other words, there is a 95% probability that the price for the effort would actually be less than $1330. Again, if we wanted a 50/50 probability of success, a reasonable price would be somewhere in the neighborhood of $1100. Since this is firm fixed price, if the effort actually came in at $1100, the delta of $230 between the $1100 and the $1330 price would represent additional profit to the contractor.

This process of identifying distributions for each of our assumptions provides a number of advantages over the simple point estimate. One, the variability in the total distribution not only communicates a sense of risk, but also suggests which contract type might be appropriate – low variability, low risk, firm fixed price; moderate variability, moderate risk, fixed price incentive; high variability, high risk, cost plus. Second, the process of identifying the distributions gives us insight into the risk drivers. We can vary our assumptions, rerun the simulation, and assess the impact on the total distribution (what we might call sensitivity analysis). Third, the process provides us a better understanding of the requirements and the statement of work (SOW), regardless of whether it was the contractor or the government that generated the SOW. It provides us the means of communicating with the contractor to truly determine whether we have a mutual understanding of the task at hand.

The IPT is an ideal setting for the use of risk analysis since by definition it includes representation from various functional specialties, not only in the government, but potentially from the contractor as well. The use of an IPT in risk management is further discussed in the extract from the Risk Management Guide which is included later in this chapter.
LEARNING OBJECTIVES

- TLO: Given a point estimate (contract cost, ceiling price) analyze associated Cost Risks
  - ELO 1: Describe characteristics of the three approaches to assessing cost risk.
  - ELO 2: Identify the characteristics of five probability distributions
  - ELO 3: Analyze the probability of under-running or over-running a given point estimate
ESTIMATING MOST LIKELY COST

<table>
<thead>
<tr>
<th>Cost Element</th>
<th>Cost Estimate</th>
<th>Likely Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controls &amp; Displays</td>
<td>$200.00</td>
<td>Most Likely</td>
</tr>
<tr>
<td>Receiver</td>
<td>$200.00</td>
<td>Most Likely</td>
</tr>
<tr>
<td>Transmitter</td>
<td>$200.00</td>
<td>Most Likely</td>
</tr>
<tr>
<td>Processor</td>
<td>$200.00</td>
<td>Most Likely</td>
</tr>
<tr>
<td>Power Supply</td>
<td>$200.00</td>
<td>Most Likely</td>
</tr>
<tr>
<td>Total</td>
<td>$1000.00</td>
<td></td>
</tr>
</tbody>
</table>

NORMAL DISTRIBUTION

Mean Median Mode
### ESTIMATING MOST LIKELY COST

<table>
<thead>
<tr>
<th>Cost Element</th>
<th>Cost Estimate</th>
<th>Low $100, High $500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controls &amp; Displays</td>
<td>$200.00</td>
<td></td>
</tr>
<tr>
<td>Receiver</td>
<td>$200.00</td>
<td></td>
</tr>
<tr>
<td>Transmitter</td>
<td>$200.00</td>
<td></td>
</tr>
<tr>
<td>Processor</td>
<td>$200.00</td>
<td></td>
</tr>
<tr>
<td>Power Supply</td>
<td>$200.00</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>??</strong></td>
<td><strong>Most Likely</strong></td>
</tr>
</tbody>
</table>

### ESTIMATING MOST LIKELY COST

- Controls & Displays: $200.00
- Receiver: $200.00
- Transmitter: $200.00
- Processor: $200.00
- Power Supply: $200.00
- **Total**: **??**

Diagram showing cost distribution between $100 and $500.
## ESTIMATING MOST LIKELY COST

<table>
<thead>
<tr>
<th>Cost Element</th>
<th>Cost Estimate</th>
<th>Cost Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controls &amp; Displays</td>
<td>Sample</td>
<td>Sample</td>
</tr>
<tr>
<td>Receiver</td>
<td>Sample</td>
<td>Sample</td>
</tr>
<tr>
<td>Transmitter</td>
<td>Sample</td>
<td>Sample</td>
</tr>
<tr>
<td>Processor</td>
<td>Sample</td>
<td>Sample</td>
</tr>
<tr>
<td>Power Supply</td>
<td>Sample</td>
<td>Sample</td>
</tr>
<tr>
<td>Total</td>
<td>Most Likely?</td>
<td>$$$</td>
</tr>
</tbody>
</table>
CPRG – VOLUME 4, CHAPTER 1

- Contract type selection is the principal method of allocating cost risk between the Government and the contractor. As you match contract type to contract risk, consider the following:
  - Identify available contract types;
  - Consider acquisition method;
  - Consider commerciality of the requirement;
  - **Consider cost risk associated with the contract action;**
  - Consider appropriate performance incentives;
  - Consider the accounting system adequacy; and
  - Document the selection decision.

COST RISK AND CONTRACT TYPES

- Cost estimates, whether they are the offeror’s proposed or the Government’s recommended, are point estimates. In all contracts involving forward pricing, the point estimate is a projection of what the estimator believes is most likely to happen. Since things rarely happen exactly as predicted, there is usually some variation between projected and actual cost. The greater the potential variability between the projected and actual cost, the greater the cost risk.
COST RISK AND CONTRACT TYPES

- As a minimum, your appraisal of cost risk should consider two areas of particular concern, contract performance risk and market risk.
- **Performance Risk.** Most contract cost risk is related to contract requirements and the uncertainty surrounding contract performance. The lower the uncertainty the lower the risk. Therefore, your appraisal of cost risk should begin with an appraisal of performance risk. For larger more complex contracts, you will likely need assistance from other members of the Government Acquisition Team (e.g., representatives from the requiring activity, engineering staff, contracting, and program/project management).
  - Areas that you consider should include:
  - Stability and clarity of the contract specifications or statement of work;
  - Type and complexity of the item or service being purchased;
  - Availability of historical pricing data;
  - Prior experience in providing required supplies or services;
  - Urgency of the requirement;
  - Contractor technical capability and financial responsibility; and
  - Extent and nature of proposed subcontracting.

COST RISK AND CONTRACT TYPES

- Performance risk should be reduced from a high to a relatively low level, as the requirement progresses from vague to well-defined and experience with the product increases.
- Research and development contracts generally have a rather high performance risk. This is due to the factor of ill-defined requirements that arise from the necessity to deal beyond, or at least very near, the upper limits of current technology (i.e., "the state of the art").
- Follow-on production contracts generally have a relatively low performance risk. Requirements are well known, there is a cost history to draw on, contractors have experience producing the product, etc.
- As performance risk changes, so should contract type. Note that cost-reimbursement, time & materials, or labor-hour contracts are generally associated with higher-risk requirements and fixed-price contracts are generally associated with lower-risk requirements.
COST RISK AND CONTRACT TYPES

- **Market Risk.** Changes in the marketplace will also affect contract costs. Preferred acquisition practice calls for forward pricing of contract efforts, because forward pricing provides a baseline which you and the contractor can use to measure cost or price performance against contract effort. Forward pricing requires the contracting parties to make assumptions about future changes in the marketplace. A volatile market will increase the cost risk involved in contract pricing, particularly when the contract period will extend several years. What will material and labor cost two years from now? Will material shortages occur two years from now? In cases where these unknown costs are significant, contract period risk becomes an important consideration in selection of contract type.

- **Fixed-price contracts with economic price adjustment,** for example, are designed specifically to reduce this risk for contractors.

RECORD OF WEIGHTED GUIDELINES APPLICATION, DD FORM 1547

[Form image]
MEMORANDUM FOR ACQUISITION PROFESSIONALS

Sept 14, 2010

SUBJECT: Better Buying Power: Guidance for Obtaining Greater Efficiency and Productivity in Defense Spending

Increase the use of Fixed-Price Incentive Firm Target (FPIF) contract type where appropriate using a 50/50 share line and 120 percent ceiling as a point of departure...

A 50/50 share line suggests that the government and contractor have a common view of the likely contract execution cost. A 50/50 share line should represent a point where the estimate is deemed equally likely to be too low or too high. A flat or steep share line suggests that the government and contractor do not see project cost the same way. These differences in view should be discussed and considered as the basis for adjusting the target cost before an uneven share line is agreed to in contract. This might occur, for example, earlier in a program where the costs are inherently more uncertain.

INCREASE THE USE OF FIXED-PRICE INCENTIVE (FIRM TARGET) CONTRACTS, PGI 216.403-1 (3), ANALYZING RISK

- The first step is establishing a target cost for which the probability of an underrun and overrun are considered equal and therefore the risks and rewards are shared equally, hence the 50/50 share is the point of departure. Equally important is determining that the contractor has a high probability of being able to accomplish the effort within a ceiling percentage of 120%. In accomplishing both these steps, the analysis of risk is essential.

- **Analyzing Risk.** Too often risk is evaluated only in general terms without attempting to quantify the risk posed by the various elements of cost. Also, a contracting officer may incorrectly fall back on the share ratios and ceiling percentages negotiated on prior contracts or other programs, without examining the specific risks.
INCREASE THE USE OF FIXED-PRICE INCENTIVE (FIRM TARGET) CONTRACTS, PGI 216.403-1 (3), ANALYZING RISK

- Whether being used to select the proper contract type or establishing share lines and ceiling price on an FPIF contract, the analysis of risk as it pertains to the prime contractor is key. From a contractor’s perspective, all risks, including technical and schedule risk have financial ramifications. Technical and schedule risks, if realized, generally translate into increased effort which means increased cost. Therefore all risk can be translated into cost risk and quantified. Risk always has two components that must be considered in the quantification: the magnitude of the impact and the probability that it will occur.
- When cost risk is quantified, it is much easier to establish a reasonable ceiling percentage. The ceiling percentage is applicable to the target cost on the prime contract. It is important to understand the degree of risk that various cost elements pose in relation to that target cost.

WHAT IS RISK?

Risks have three components:
1. A future root cause or event (yet to happen), which, if eliminated or corrected, would prevent a potential consequence from occurring. For example, an engine stall has the consequence of a potential aircraft crash. Eliminating engine stalls would prevent an aircraft from crashing due to this cause (it could still crash from other causes, such as failure of the landing gear). So the root cause is engine stall, not an aircraft crashing.
2. A probability (or likelihood) assessed at the present time of that future root cause or event occurring. For example, during recent tests, an aircraft engine was found to stall four out of ten times whenever a certain maneuver was performed. We can assess a 40% probability that this aircraft engine will stall whenever a pilot has to perform this maneuver in the future.
3. The consequence (or effect) of that future occurrence or event if it actually occurs. Continuing with our example, if the engine stalls, the consequence would be potential loss of aircraft and loss of life (among others).
RISK AND ROOT CAUSES

- **Scope/Requirements Risk:** It identifies sensitivity of the program to changes in the threat description or to changes in the system description or requirements. As a result of these changes to the threat and/or requirements, the system configurations will change over the course of the program's development.
- **Cost Risk:** This category includes the effects of budget and affordability decisions and the effects of inherent errors in the cost estimating technique(s) used (i.e., statistical uncertainties -- data is subject to error and sampling techniques have random error that will always exist).
- **Schedule Risk:** This factor includes the effects of programmatic schedule decisions, the inherent errors in schedule estimating, and external physical constraints.
- **Technical Risk:** Identifies the degree to which the technology proposed for the program has been sufficiently demonstrated to be realistically capable of meeting all of the program's objectives. In other words, can the system be developed with the current technology we have at the time. This type of risk is highest during R&D as we typically cannot move into production until the technology has been sufficiently demonstrated.

PROBABILITIES AND CONSEQUENCES

<table>
<thead>
<tr>
<th>Likelihood</th>
<th>Consequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

23

24
APPROACHES FOR ASSESSING COST RISK

1. Sensitivity Analysis
2. Monte Carlo
3. Symmetric Approximation

SENSITIVITY ANALYSIS

• Measure how sensitive system cost is to variations in non-cost parameters
• Method of testing assumptions by adjusting cost drivers to indicate magnitude of variations
• Provides a quantitative assessment of potential changes to “cost drivers”
• Select variables and adjust one at a time based on different assumptions
SENSITIVITY ANALYSIS

Sensitivity Analysis Matrix for buying 50 Units

<table>
<thead>
<tr>
<th>Alternative</th>
<th>T1 cost</th>
<th>AUC @80% Slope</th>
<th>AUC @ 81%</th>
<th>AUC @82% slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternative A</td>
<td>$1,000.00</td>
<td>$403.00</td>
<td>$423.00</td>
<td>$443.00</td>
</tr>
<tr>
<td>Alternative B</td>
<td>$950.00</td>
<td>$383.00</td>
<td>$402.00</td>
<td>$421.00</td>
</tr>
<tr>
<td>Alternative C</td>
<td>$900.00</td>
<td>$363.00</td>
<td>$380.00</td>
<td>$399.00</td>
</tr>
</tbody>
</table>

SENSITIVITY ANALYSIS

Graph showing cost and price analysis with T1 cost and AUC values.
MONTE CARLO

Steps:
1. Identify the areas of uncertainty
2. Develop the probability distribution for each area
3. Randomly sample from each element (iteratively) and develop a distribution of the total cost

Just like the @Risk demonstration…
ITERATIONS IN MONTE CARLO SIMULATION

<table>
<thead>
<tr>
<th>Trial</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$21,542,764</td>
</tr>
</tbody>
</table>

CREATING THE TOTAL COST DISTRIBUTION

Trial | Total Cost
---    |------------
1      | $21,542,764
SYMmetric Approximation
(Poor Man’s Monte Carlo)

Steps:
1. Identify the areas of uncertainty
2. Develop the probability distribution for each area
3. Calculate the mean and variance for each distribution
4. Estimate the mean, variance & SD for the total cost

1. Identify the areas of uncertainty
   - Major elements of cost
   - Assumptions that affect major elements of cost
   - Understanding of contractor’s basis of estimate
   - Developing the government objective
   - Personal experience
   - Subject matter experts
   - Historical data
2. Develop the probability distributions
   - Data
   - Personal experience
   - Expert Opinion
CON 270, Intermediate Cost and Price Analysis

DISTRIBUTIONS

Normal
Uniform
Triangular
Discrete

Beta
Alpha = 2
Beta = 3
Alpha = 3
Beta = 3
Alpha = 3
Beta = 2

DISTRIBUTIONS

Appropriate where the analyst is completely uncertain about a given cost, except that it must be between some specified minimum and maximum values. Every value between the minimum and the maximum has equal probability of occurring.

Uniform

Probably the most widely used distribution. Requires minimum, maximum and a most likely value to be known. The most likely represents the mode of the triangle (its peak), and in pricing, would typically be the value used in the government objective.

Triangular

Discrete

Used where discrete (distinct) events or outcomes exist and the means are available to objectively or subjectively state the likelihood of each outcome occurring. The sum of the probabilities of the outcomes equals one.
DISTRIBUTIONS

Used when outcomes are equally likely to occur on either side of the average value; symmetric and continuous, unbounded, allowing for negative values. Need to be able to determine the mean and standard deviation.

Beta (right skew) Beta (normal) Beta (left skew)

Similar to the normal distribution, but does not allow for negative cost, this continuous distribution can be symmetric or skewed. The PERT beta allows the user to specify minimum and maximum values, and the shape parameters.

SYMMETRIC APPROXIMATION

3. Calculate the mean and variance of each distribution

Uniform

\[ \mu = \frac{A + B}{2} \]
\[ \sigma^2 = \frac{(B - A)^2}{12} \]

\[ A = \text{minimum} \]
\[ B = \text{maximum} \]

Triangular

\[ \mu = \frac{A + B + C}{3} \]
\[ \sigma^2 = \frac{A^3 + B^3 + C^3 - 3(A \times B)(C - A)(B - C)}{18} \]

\[ A = \text{minimum} \]
\[ B = \text{maximum} \]
\[ C = \text{mode} \]

Beta

\[ \mu = \frac{(A + \beta)(B + \alpha)}{\alpha + \beta} \]
\[ \sigma^2 = \frac{(B - A)^2}{(\alpha + \beta + 1)(\alpha + \beta)^2} \]

\[ A = \text{minimum} \]
\[ B = \text{maximum} \]

Alpha and Beta correspond to shape preference
SYMmetric APPROXIMATION

4. Estimate the mean, variance, & SD for the total cost

1) Sum the means of the cost elements to get $\mu_{TC}$ (mean of the total cost)
2) Sum the variances of the cost elements to get $\sigma^2_{TC}$ (variance of the total cost)
3) Calculate the square root of $\sigma^2_{TC}$ to arrive at the $\sigma_{TC}$ (standard deviation of the TC)

CORRELATION – HOW COSTS ARE RELATED

In our previous example we treated the elements of cost as if they were independent of each other. In other words, the cost of the Transmitter had no bearing on the cost of the Processor or Power Supply.

But what if it did? What if the range on the Transmitter had a bearing on the demands on the Processor and the amount of power required by the Power Supply.

In that case we would say that these components (and their costs) were correlated (i.e. a change in one would result in a change in the other).
CORRELATION – THE BOTTOM LINE

In order to understand how correlation affects the bottom line let's take the example of direct manufacturing labor and manufacturing overhead.

Since overhead is a function of the direct costs, we would say that they are correlated. If there was an increase in our estimate of direct labor, we would also increase our estimate of the overhead. Not doing so would understate the total cost.

Likewise, if our objective for direct labor decreased, then we would also decrease our estimate of overhead, and consequently reduce the total cost.

CORRELATION – THE BOTTOM LINE

You can see that if we treated the direct and indirect costs as independent, we would be understating both the lower and upper bounds of the total cost. Or put another way, understating the variation in the total cost.

While some aspects of correlation can be incorporated into a Symmetric Approximation, one of the reasons that practitioners prefer using Monte Carlo applications is because they provide a much more robust treatment of correlation, and consequently a much more accurate depiction of the variation in the total cost distribution.
### SYMMETRIC APPROXIMATION

#### PRACTICE EXERCISE

**From the beginning our the Lesson...**

<table>
<thead>
<tr>
<th>Element</th>
<th>Low</th>
<th>Most Likely</th>
<th>High</th>
<th>Mean</th>
<th>Variance</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controls &amp; Displays</td>
<td>100</td>
<td>200</td>
<td>500</td>
<td>266.67</td>
<td>?</td>
<td>Triangular</td>
</tr>
<tr>
<td>Receiver</td>
<td>100</td>
<td>200</td>
<td>500</td>
<td>266.67</td>
<td>?</td>
<td>Triangular</td>
</tr>
<tr>
<td>Transmitter</td>
<td>100</td>
<td>200</td>
<td>500</td>
<td>266.67</td>
<td>?</td>
<td>Triangular</td>
</tr>
<tr>
<td>Processor</td>
<td>100</td>
<td>200</td>
<td>500</td>
<td>266.67</td>
<td>?</td>
<td>Triangular</td>
</tr>
<tr>
<td>Power Supply</td>
<td>100</td>
<td>200</td>
<td>500</td>
<td>266.67</td>
<td>?</td>
<td>Triangular</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>500</td>
<td>1000</td>
<td>2500</td>
<td>1333</td>
<td>?</td>
<td>Triangular</td>
</tr>
</tbody>
</table>

\[ \text{Mean}(\mu) = \frac{A + B + C}{3} = \frac{100 + 500 + 200}{3} = 266.67 \]

---

**SYMMETRIC APPROXIMATION

#### PRACTICE EXERCISE**

<table>
<thead>
<tr>
<th>Element</th>
<th>Low</th>
<th>Most Likely</th>
<th>High</th>
<th>Mean</th>
<th>Variance</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controls &amp; Displays</td>
<td>100</td>
<td>200</td>
<td>500</td>
<td>266.67</td>
<td>7222.22</td>
<td>Triangular</td>
</tr>
<tr>
<td>Receiver</td>
<td>100</td>
<td>200</td>
<td>500</td>
<td>266.67</td>
<td>7222.22</td>
<td>Triangular</td>
</tr>
<tr>
<td>Transmitter</td>
<td>100</td>
<td>200</td>
<td>500</td>
<td>266.67</td>
<td>7222.22</td>
<td>Triangular</td>
</tr>
<tr>
<td>Processor</td>
<td>100</td>
<td>200</td>
<td>500</td>
<td>266.67</td>
<td>7222.22</td>
<td>Triangular</td>
</tr>
<tr>
<td>Power Supply</td>
<td>100</td>
<td>200</td>
<td>500</td>
<td>266.67</td>
<td>7222.22</td>
<td>Triangular</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>500</td>
<td>1000</td>
<td>2500</td>
<td>1333</td>
<td>36111</td>
<td>Triangular</td>
</tr>
</tbody>
</table>

\[ \text{Variance}(\sigma^2) = \frac{A^2 + B^2 + C^2 - (A \times B) - (A \times C) - (B \times C)}{18} \]
\[ = \frac{100^2 + 500^2 + 200^2 - (100 \times 500) - (100 \times 200) - (500 \times 200)}{18} \]
\[ = \frac{10,000 + 250,000 + 40,000 - 50,000 - 20,000 - 100,000}{18} \]
\[ = \frac{130,000}{18} = 7222.22 \]

**Standard Deviation** \( \sigma_T = \sqrt{\sigma^2} = \sqrt{36111} = 190 \)
Radar Modification Risk Analysis Exercise

Your organization has been negotiating with the original manufacturer for a modification to their radar system. The program manager and her staff have been working the proposal using an IPT approach. You have been provided with the agreed-to positions for the elements of cost and are now trying to determine the amount of risk inherent in the effort so that you can assign the appropriate risk factors on the weighted guidelines.

You have gone back to the government focal points for the various elements of cost and conducted interviews that yielded the following information:

1. The audit on direct materials indicated a likely range between $42,250 and $47,395.

2. Given the estimates of design engineering labor and the forward pricing rate agreement (FPRA), the most likely engineering direct labor cost is $175,500. Based on the assessment, the cost could be as low as $165,250 or as high as $197,500.

3. The engineering overhead rate based on the FPRA is 175% and the distribution for overhead would correspond to the distribution of direct engineering labor.

4. Your engineer is fairly confident in the estimate of manufacturing hours. The primary cost driver in manufacturing direct labor costs is the ongoing company-union labor negotiation. Speaking with company representatives, they believe there is a 3-out-of-4 chance the company will prevail in negotiations, in which case the direct manufacturing cost would be $225,000. If the union prevails the cost would be $245,125.

5. The manufacturing overhead rate based on the FPRA is 215% and the distribution for overhead would correspond to the distribution of direct manufacturing labor.

6. The other direct costs (ODCs) are based on the average price of special tooling equipment. DCMA located eight current prices for this type of tooling: $11,195; $12,950; $11,750; $12,350; $12,500; $12,450; $12,550; $13,450.

7. Test engineering is treated as a separate department. The test manager said that while the costs based on the hours and FPRA could range between $25,000 and $37,250 that he expected it more likely that the hours would be in the lower end of that range. He said that estimates of test hours tend to be right skewed.

Develop the distributions for each element of cost and estimate the total cost and variance for subtotal costs.
Probability under a Normal Distribution

The area under a normal distribution is equal to 1.0000. Since the curve is symmetrical, there is .5000 in each half.

The distance between the mean of any normal distribution (μ) and a value of X can be measured in standard deviations (Z). The Z table indicates the probability or area between 0 and Z standard deviations. Since “μ” is in the middle, it is at zero (0) standard deviations. The only thing we need to determine is how many standard deviations (Z) that X is from μ.

\[ Z = \frac{X - \mu}{\sigma} \]

The Z value is calculated as: \( Z = \frac{X - \mu}{\sigma} \)

After calculating the Z value, we can look-up the probability between 0 and Z standard deviations in a Z table. In a Z table the numbers down the left hand column, combined with the numbers across the top, represent standard deviations. The values within the table represent the areas between 0 and Z standard deviations.

<table>
<thead>
<tr>
<th>Z</th>
<th>.00</th>
<th>.01</th>
<th>.02</th>
<th>.03</th>
<th>.04</th>
<th>.05</th>
<th>.06</th>
<th>.07</th>
<th>.08</th>
<th>.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>.0000</td>
<td>.0040</td>
<td>.0080</td>
<td>.0120</td>
<td>.0160</td>
<td>.0199</td>
<td>.0239</td>
<td>.0279</td>
<td>.0319</td>
<td>.0359</td>
</tr>
<tr>
<td>0.10</td>
<td>.0398</td>
<td>.0438</td>
<td>.0478</td>
<td>.0517</td>
<td>.0557</td>
<td>.0596</td>
<td>.0636</td>
<td>.0675</td>
<td>.0714</td>
<td>.0753</td>
</tr>
<tr>
<td>0.20</td>
<td>.0793</td>
<td>.0832</td>
<td>.0871</td>
<td>.0910</td>
<td>.0948</td>
<td>.0987</td>
<td>.1026</td>
<td>.1064</td>
<td>.1103</td>
<td>.1141</td>
</tr>
<tr>
<td>0.30</td>
<td>.1179</td>
<td>.1217</td>
<td>.1255</td>
<td>.1293</td>
<td>.1331</td>
<td>.1368</td>
<td>.1406</td>
<td>.1443</td>
<td>.1480</td>
<td>.1517</td>
</tr>
<tr>
<td>0.40</td>
<td>.1554</td>
<td>.1591</td>
<td>.1628</td>
<td>.1664</td>
<td>.1700</td>
<td>.1736</td>
<td>.1772</td>
<td>.1808</td>
<td>.1844</td>
<td>.1879</td>
</tr>
</tbody>
</table>

Let’s say that the average (μ) age of people in the United States is 40 with a standard deviation (σ) of 10. What percentage of the population is between 40 and 45 years of age?

\[ Z = \frac{X - \mu}{\sigma} = \frac{45 - 40}{10} = .5 \]

\( .5 \) standard deviations

We go down the left hand column to .50 and across to the column of .00 (the .00 column indicates that the second decimal place in the Z calculation of .50 is a 0). We locate a probability of .1915. This could be stated as 19.15% of the population is between 40 and 45 years of age. Graphically, it is represented by the gray area in the picture.

If we wanted to know the percentage of the population over 45, it would be the area to the right of the gray area. Since the area on the right side of the curve is equal to .5000, if we subtracted the gray area (.1915) from .5000, it would leave us with the area to the right.

\[ .5000 - .1915 = .3085 \] This could be stated as 30.85% of the population is over 45 years old.

If we wanted to know the percentage of the population under 45, we would add the gray area (.1915) to the area in the left half of the curve (.5000).

\[ .5000 + .1915 = .6915 \] This could be stated as 69.15% of the population is under 45 years old.
What if we wanted to know the percentage of the population between 35 and 45 years of age? Graphically the problem would look like this:

In addition to our previous calculations, we would need to determine the probability of people being between 35 and 40 years of age.

\[
Z = \frac{X - \mu}{\sigma} = \frac{35 - 40}{10} = -0.5 \text{ standard deviations}
\]

When referencing probability in a Z table we treat the Z as an absolute value, so we interpret -0.5 as 0.5 standard deviations. We want to know the probability between 0 and 0.5 standard deviations. Once again, referencing 0.5 standard deviations in the Z table gives us a probability of 0.1915.

In this case we want to add the two gray areas.

\[0.1915 + 0.1915 = 0.3830\]  Stated as 38.30% of the population is between 35 and 45 years of age.

What if we wanted to know the percentage of the population between 42 and 45 years of age? Graphically the problem would look like this:

Since we already know that the probability between 40 and 45 is 0.1915, we just need to determine the probability of people being between 40 and 42 years of age.

\[
Z = \frac{X - \mu}{\sigma} = \frac{42 - 40}{10} = 0.2 \text{ standard deviations}
\]

Referencing 0.2 standard deviations on the Z table we locate a probability of 0.0793.

The shaded area in our diagram represents the difference between 0.50 and 0.20 standard deviations, so we subtract the probabilities.

\[0.1915 - 0.0793 = 0.1122\]  Stated as 11.22% of the population is between 42 and 45 years of age.

---

So, what process should we use to determine probabilities under a Normal Distribution?

1. Draw a diagram of the area you are trying to determine.
   ![Diagram]

2. Calculate the Z value(s) \[Z = \frac{X - \mu}{\sigma}\]

3. Locate the probability in the Z table associated with the Z (standard deviations) you calculated.

4. Determine whether the probability you located in the Z table represents the area in your diagram, or whether an addition or subtraction will be required.
Symmetric Approximation

**Step 1** – Identify the areas of uncertainty in your estimate (labor hours, material prices, etc.)

**Step 2** – Select a distribution type for each area of uncertainty you are going to quantify

**Normal**

**Uniform**

**Triangular**

**Beta**

**Step 3** – Calculate the Mean and Variance for each cost element you are analyzing

### Uniform

\[ \mu = \frac{A + B}{2} \]

\[ \sigma^2 = \frac{(B - A)^2}{12} \]

### Triangular

\[ \mu = \frac{A + B + C}{3} \]

\[ \sigma^2 = \frac{A^2 + B^2 + C^2 - (A \times B) - (A \times C) - (B \times C)}{18} \]

### Beta

\[ \mu = \frac{(A \times \beta) + (B \times \alpha)}{\alpha + \beta} \]

\[ \sigma^2 = \frac{(\alpha \times \beta) \times (B - A)}{(\alpha + \beta + 1) \times (\alpha + \beta)^2} \]
Intermediate Cost and Price Analysis

Alpha and Beta correspond to shape preference.
**Step 4** – Calculate the Mean and Standard Deviation for the Total Cost (TC)

1) Sum the means of the cost elements to get $\mu_{TC}$ (mean of the total cost)

2) Sum the variances of the cost elements to get $\sigma^2_{TC}$ (variance of the total cost)

3) Calculate the square root of $\sigma^2_{TC}$ to arrive at the $\sigma_{TC}$ (standard deviation of the TC)

**Step 5** – (For example) Determine the probability of Over-Running the Gov’t objective

Distribution of Total System Cost
(Normal Distribution centered at $\mu_{TC}$)

$$Z = \frac{(Gov't - \mu_{TC})}{\sigma_{TC}}$$

<table>
<thead>
<tr>
<th>$z$</th>
<th>.00</th>
<th>.01</th>
<th>.02</th>
<th>.03</th>
<th>.04</th>
<th>.05</th>
<th>.06</th>
<th>.07</th>
<th>.08</th>
<th>.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>.0000</td>
<td>.0040</td>
<td>.0080</td>
<td>.0120</td>
<td>.0160</td>
<td>.0199</td>
<td>.0239</td>
<td>.0279</td>
<td>.0319</td>
<td>.0359</td>
</tr>
<tr>
<td>0.10</td>
<td>.0398</td>
<td>.0438</td>
<td>.0478</td>
<td>.0517</td>
<td>.0557</td>
<td>.0596</td>
<td>.0636</td>
<td>.0675</td>
<td>.0714</td>
<td>.0753</td>
</tr>
<tr>
<td>0.20</td>
<td>.0793</td>
<td>.0832</td>
<td>.0871</td>
<td>.0910</td>
<td>.0948</td>
<td>.0987</td>
<td>.1026</td>
<td>.1064</td>
<td>.1103</td>
<td>.1141</td>
</tr>
<tr>
<td>0.30</td>
<td>.1179</td>
<td>.1217</td>
<td>.1255</td>
<td>.1293</td>
<td>.1331</td>
<td>.1368</td>
<td>.1406</td>
<td>.1443</td>
<td>.1480</td>
<td>.1517</td>
</tr>
<tr>
<td>0.40</td>
<td>.1554</td>
<td>.1591</td>
<td>.1628</td>
<td>.1664</td>
<td>.1700</td>
<td>.1736</td>
<td>.1772</td>
<td>.1808</td>
<td>.1844</td>
<td>.1879</td>
</tr>
<tr>
<td>0.60</td>
<td>.2257</td>
<td>.2291</td>
<td>.2324</td>
<td>.2357</td>
<td>.2389</td>
<td>.2422</td>
<td>.2454</td>
<td>.2486</td>
<td>.2517</td>
<td>.2549</td>
</tr>
<tr>
<td>0.70</td>
<td>.2580</td>
<td>.2611</td>
<td>.2642</td>
<td>.2673</td>
<td>.2704</td>
<td>.2734</td>
<td>.2764</td>
<td>.2794</td>
<td>.2823</td>
<td>.2852</td>
</tr>
<tr>
<td>0.80</td>
<td>.2881</td>
<td>.2910</td>
<td>.2939</td>
<td>.2967</td>
<td>.2995</td>
<td>.3023</td>
<td>.3051</td>
<td>.3078</td>
<td>.3106</td>
<td>.3133</td>
</tr>
<tr>
<td>0.90</td>
<td>.3159</td>
<td>.3186</td>
<td>.3212</td>
<td>.3238</td>
<td>.3264</td>
<td>.3289</td>
<td>.3315</td>
<td>.3340</td>
<td>.3365</td>
<td>.3389</td>
</tr>
<tr>
<td>1.00</td>
<td>.3413</td>
<td>.3438</td>
<td>.3461</td>
<td>.3485</td>
<td>.3508</td>
<td>.3531</td>
<td>.3554</td>
<td>.3577</td>
<td>.3599</td>
<td>.3621</td>
</tr>
</tbody>
</table>
The Standard Normal Distribution (Z Table)

<table>
<thead>
<tr>
<th>z</th>
<th>.00</th>
<th>.01</th>
<th>.02</th>
<th>.03</th>
<th>.04</th>
<th>.05</th>
<th>.06</th>
<th>.07</th>
<th>.08</th>
<th>.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>.0000</td>
<td>.0040</td>
<td>.0080</td>
<td>.0120</td>
<td>.0160</td>
<td>.0199</td>
<td>.0239</td>
<td>.0279</td>
<td>.0319</td>
<td>.0359</td>
</tr>
<tr>
<td>0.10</td>
<td>.0398</td>
<td>.0438</td>
<td>.0478</td>
<td>.0517</td>
<td>.0557</td>
<td>.0596</td>
<td>.0636</td>
<td>.0675</td>
<td>.0714</td>
<td>.0753</td>
</tr>
<tr>
<td>0.20</td>
<td>.0793</td>
<td>.0832</td>
<td>.0871</td>
<td>.0910</td>
<td>.0948</td>
<td>.0987</td>
<td>.1026</td>
<td>.1064</td>
<td>.1103</td>
<td>.1141</td>
</tr>
<tr>
<td>0.30</td>
<td>.1179</td>
<td>.1217</td>
<td>.1255</td>
<td>.1293</td>
<td>.1331</td>
<td>.1368</td>
<td>.1406</td>
<td>.1443</td>
<td>.1480</td>
<td>.1517</td>
</tr>
<tr>
<td>0.40</td>
<td>.1554</td>
<td>.1591</td>
<td>.1628</td>
<td>.1664</td>
<td>.1700</td>
<td>.1736</td>
<td>.1772</td>
<td>.1808</td>
<td>.1844</td>
<td>.1879</td>
</tr>
<tr>
<td>0.60</td>
<td>.2257</td>
<td>.2291</td>
<td>.2324</td>
<td>.2357</td>
<td>.2389</td>
<td>.2422</td>
<td>.2454</td>
<td>.2486</td>
<td>.2517</td>
<td>.2549</td>
</tr>
<tr>
<td>0.70</td>
<td>.2580</td>
<td>.2611</td>
<td>.2642</td>
<td>.2673</td>
<td>.2704</td>
<td>.2734</td>
<td>.2764</td>
<td>.2794</td>
<td>.2823</td>
<td>.2852</td>
</tr>
<tr>
<td>0.80</td>
<td>.2881</td>
<td>.2910</td>
<td>.2939</td>
<td>.2967</td>
<td>.2995</td>
<td>.3023</td>
<td>.3051</td>
<td>.3078</td>
<td>.3106</td>
<td>.3133</td>
</tr>
<tr>
<td>0.90</td>
<td>.3159</td>
<td>.3186</td>
<td>.3212</td>
<td>.3238</td>
<td>.3264</td>
<td>.3289</td>
<td>.3315</td>
<td>.3340</td>
<td>.3365</td>
<td>.3389</td>
</tr>
<tr>
<td>1.00</td>
<td>.3413</td>
<td>.3438</td>
<td>.3461</td>
<td>.3485</td>
<td>.3508</td>
<td>.3531</td>
<td>.3554</td>
<td>.3577</td>
<td>.3599</td>
<td>.3621</td>
</tr>
<tr>
<td>1.10</td>
<td>.3643</td>
<td>.3665</td>
<td>.3686</td>
<td>.3708</td>
<td>.3729</td>
<td>.3749</td>
<td>.3770</td>
<td>.3790</td>
<td>.3810</td>
<td>.3830</td>
</tr>
<tr>
<td>1.20</td>
<td>.3849</td>
<td>.3869</td>
<td>.3888</td>
<td>.3907</td>
<td>.3925</td>
<td>.3944</td>
<td>.3962</td>
<td>.3980</td>
<td>.3997</td>
<td>.4015</td>
</tr>
<tr>
<td>1.30</td>
<td>.4032</td>
<td>.4049</td>
<td>.4066</td>
<td>.4082</td>
<td>.4099</td>
<td>.4115</td>
<td>.4131</td>
<td>.4147</td>
<td>.4162</td>
<td>.4177</td>
</tr>
<tr>
<td>1.40</td>
<td>.4192</td>
<td>.4207</td>
<td>.4222</td>
<td>.4236</td>
<td>.4251</td>
<td>.4265</td>
<td>.4279</td>
<td>.4292</td>
<td>.4306</td>
<td>.4319</td>
</tr>
<tr>
<td>1.50</td>
<td>.4332</td>
<td>.4345</td>
<td>.4357</td>
<td>.4370</td>
<td>.4382</td>
<td>.4394</td>
<td>.4406</td>
<td>.4418</td>
<td>.4429</td>
<td>.4441</td>
</tr>
<tr>
<td>1.60</td>
<td>.4452</td>
<td>.4463</td>
<td>.4474</td>
<td>.4484</td>
<td>.4495</td>
<td>.4505</td>
<td>.4515</td>
<td>.4525</td>
<td>.4535</td>
<td>.4545</td>
</tr>
<tr>
<td>1.70</td>
<td>.4554</td>
<td>.4564</td>
<td>.4573</td>
<td>.4582</td>
<td>.4591</td>
<td>.4599</td>
<td>.4608</td>
<td>.4616</td>
<td>.4625</td>
<td>.4633</td>
</tr>
<tr>
<td>1.80</td>
<td>.4641</td>
<td>.4649</td>
<td>.4656</td>
<td>.4664</td>
<td>.4671</td>
<td>.4678</td>
<td>.4686</td>
<td>.4693</td>
<td>.4699</td>
<td>.4706</td>
</tr>
<tr>
<td>1.90</td>
<td>.4713</td>
<td>.4719</td>
<td>.4726</td>
<td>.4732</td>
<td>.4738</td>
<td>.4744</td>
<td>.4750</td>
<td>.4756</td>
<td>.4761</td>
<td>.4767</td>
</tr>
<tr>
<td>2.00</td>
<td>.4772</td>
<td>.4778</td>
<td>.4783</td>
<td>.4788</td>
<td>.4793</td>
<td>.4798</td>
<td>.4803</td>
<td>.4808</td>
<td>.4812</td>
<td>.4817</td>
</tr>
<tr>
<td>2.10</td>
<td>.4821</td>
<td>.4826</td>
<td>.4830</td>
<td>.4834</td>
<td>.4838</td>
<td>.4842</td>
<td>.4846</td>
<td>.4850</td>
<td>.4854</td>
<td>.4857</td>
</tr>
<tr>
<td>2.20</td>
<td>.4861</td>
<td>.4864</td>
<td>.4868</td>
<td>.4871</td>
<td>.4875</td>
<td>.4878</td>
<td>.4881</td>
<td>.4884</td>
<td>.4887</td>
<td>.4890</td>
</tr>
<tr>
<td>2.30</td>
<td>.4893</td>
<td>.4896</td>
<td>.4898</td>
<td>.4901</td>
<td>.4904</td>
<td>.4906</td>
<td>.4909</td>
<td>.4911</td>
<td>.4913</td>
<td>.4916</td>
</tr>
<tr>
<td>2.40</td>
<td>.4918</td>
<td>.4920</td>
<td>.4922</td>
<td>.4925</td>
<td>.4927</td>
<td>.4929</td>
<td>.4931</td>
<td>.4932</td>
<td>.4934</td>
<td>.4936</td>
</tr>
<tr>
<td>2.50</td>
<td>.4938</td>
<td>.4940</td>
<td>.4941</td>
<td>.4943</td>
<td>.4945</td>
<td>.4946</td>
<td>.4948</td>
<td>.4949</td>
<td>.4951</td>
<td>.4952</td>
</tr>
<tr>
<td>2.60</td>
<td>.4953</td>
<td>.4955</td>
<td>.4956</td>
<td>.4957</td>
<td>.4959</td>
<td>.4960</td>
<td>.4961</td>
<td>.4962</td>
<td>.4963</td>
<td>.4964</td>
</tr>
<tr>
<td>2.70</td>
<td>.4965</td>
<td>.4966</td>
<td>.4967</td>
<td>.4968</td>
<td>.4969</td>
<td>.4970</td>
<td>.4971</td>
<td>.4972</td>
<td>.4973</td>
<td>.4974</td>
</tr>
<tr>
<td>2.80</td>
<td>.4974</td>
<td>.4975</td>
<td>.4976</td>
<td>.4977</td>
<td>.4977</td>
<td>.4978</td>
<td>.4979</td>
<td>.4979</td>
<td>.4980</td>
<td>.4981</td>
</tr>
<tr>
<td>2.90</td>
<td>.4981</td>
<td>.4982</td>
<td>.4982</td>
<td>.4983</td>
<td>.4984</td>
<td>.4984</td>
<td>.4985</td>
<td>.4985</td>
<td>.4986</td>
<td>.4986</td>
</tr>
<tr>
<td>3.00</td>
<td>.4987</td>
<td>.4987</td>
<td>.4987</td>
<td>.4988</td>
<td>.4988</td>
<td>.4988</td>
<td>.4989</td>
<td>.4989</td>
<td>.4989</td>
<td>.4990</td>
</tr>
</tbody>
</table>